Lesson 7-6: Circles and Arcs

One of the most common geometric shapes

Circles are everywhere you look. Unless you walk to school, you come to school in the morning on a circle…well, actually four circles. Some of the greatest advances in human history are based on using circles.

I'm sure all of you are familiar with the basic parts of a circle: center, radius and diameter. Today we are going to learn about some other parts of a circle and how to measure them. Tomorrow we will apply this to finding the area of circles and sectors of circles.

Naming and notation

We name a circle by its center (capital letter). The symbol for a circle is an O with a dot in the middle: $\odot$

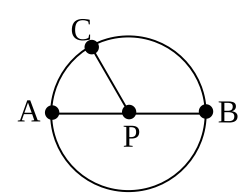
Thus we will reference circle P as $\odot P$.

Congruence

We know what it means to say that two triangles are congruent. But, what does it mean if we say two circles are congruent? How would we define this? Well, congruent figures means the figures are the same size and shape…they'd be identical if we laid one on top of the other. Well, what is a defining attribute of a circle? Is there one thing we could identify that completely describes a circle?

Given that a circle is the set of all points equidistant from a given point (the center), the distance from the center to the set of points seems like the attribute we are after. Obviously this is the radius of the circle.

We say that **circles are congruent if their radii are congruent**.



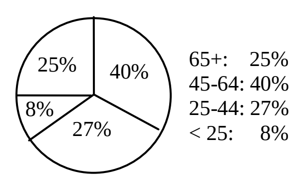
Central angle

A **central angle** is an angle whose vertex is the center of the circle.

In this figure, $\angle CPA$ is a central angle.

Circle graphs example

Central angles are often used in pie charts or **circle graphs**. Circle graphs are graphical ways of showing percentage breakdown. Here is a circle graph showing the breakdown by age of the member's of a club. For each age category, find the measure of the central angle it represents.



The 65 and older group is 25% of the pie. All the way around the pie is 360°. So the 65 and older group is 25% of 360 or 90. The 45-64 age group is 40% of 360 or 144. The 25-44 age group is 27% of 360 or 97.2. Finally, the under 25 age group is 8% of 360 or 28.8.

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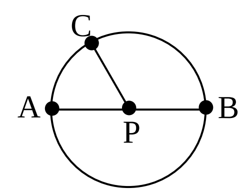
Arcs

An **<u>arc</u>** is a part of a circle around the edge. To be precise, it consists of two points on the circle and the unbroken part of the circle between the two points. Do you remember what we call an arc that goes half way around a circle? It is a **<u>semicircle</u>**. Its endpoints are the endpoints of the diameter. The symbol for an arc AC is $\overset{\frown}{AC}$. We measure an arc by its center angle. The center angle associated with $\overset{\frown}{AC}$ is $\angle APC$.



How would you name a semicircle? Would it be enough to reference by just the end points? No it wouldn't; it would be ambiguous. Which way around the circle do you mean? There are two directions you can go from one endpoint to the other. To name a semicircle, you need to name the end points and one more point. In this circle, you could name $\overset{\frown}{ACB}$ as a semicircle.

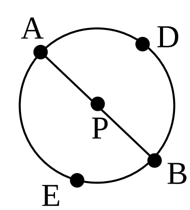
Minor and major arcs

A **<u>minor arc</u>** is an arc that is smaller than a semicircle. Thus a minor arc has measure less than 180. You can name a minor arc with just two endpoints…there is no ambiguity. In the figure above, both $\overset{\frown}{AC}$ and $\overset{\frown}{CB}$ are minor arcs.

A **<u>major arc</u>** is an arc that is larger than a semicircle. In geometry, we normally measure angles that are 180 or less. The measure of a major arc is 360 – measure of the related minor arc. Using the figure above, the measure of $\overset{\frown}{ABC}$ ($m\overset{\frown}{ABC}$) is 360 - $m\overset{\frown}{AC}$.

Minor/major arc example

Identify the minor arcs, major arcs and semicircles in $\odot P$ with A as an endpoint.



Minor arcs: $\overset{\frown}{AD}, \overset{\frown}{AE}$

Major arcs: $\overset{\frown}{ADE}(or\ \overset{\frown}{ABE}), \overset{\frown}{AED}(or\ \overset{\frown}{ABD})$

Semicircles: $\overset{\frown}{AEB}$, $\overset{\frown}{ADB}$

Adding arcs

<u>Adjacent arcs</u> are arcs that share <u>exactly</u> one point…an end point. There is no overlap. So from the example just above, $\overset{\frown}{AD}$ & $\overset{\frown}{DB}$ are adjacent arcs.

You can add the measures of adjacent arcs just as you can add adjacent angles. This leads to…

Postulate 7-1 The Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the adjacent arcs. From the example above $m\overset{\frown}{ADB} = m\overset{\frown}{AD} + m\overset{\frown}{DB}$.

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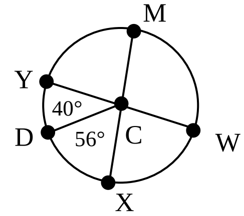
Arc addition example

Find $m\overset{\frown}{XY}$ & $m\overset{\frown}{DXM}$ in $\odot C$.



$m\overset{\frown}{XY} = m\overset{\frown}{XD} + m\overset{\frown}{DY} = 56 + 40 = 96$

Note that $\overline{XM}$ is a diameter of $\odot C$.

$m\overset{\frown}{XWM} = m\angle XWM = 180$

$m\overset{\frown}{DXM} = m\overset{\frown}{DX} + m\overset{\frown}{XWM} = 56 + 180 = 236$

Theorem 7-13 Circumference of a Circle

The **circumference** (the distance around the circle) of a circle *C*, with diameter *d* is π times the diameter. $C = \pi d \text{ or } C = 2\pi r$.

The number π is the ratio of the circumference to the diameter of any circle.

Concentric circles

Concentric circles are co-planar circles that share the same center. A target bulls-eye is an example of concentric circles.

Circumference example

A circular swimming pool with a 16*ft* diameter will be enclosed in a circular fence 4*ft* from the pool. What length of fencing material is needed? Round to the nearest whole number.

The fence will be a concentric circle with the pool. Its radius will be 4*ft* longer than the radius of the pool. The radius of the pool is 8 ($r = d \div 2 = 16 \div 2$) so the radius of the fenced circle is 12. Thus the circumference of the fenced circle is $C = 2\pi r = 2\pi 12 = 24\pi \approx 24 \bullet 3.14 = 75.36 \approx 75\, ft$ (rounded to the nearest whole #).

Arc length and arc measurement

We learned that the measure of an arc is in degrees and is determined by its central angle. We can also talk about the length of an arc. **Arc length** is a fraction of the circle's circumference; it is a measure of what part of the circumference it takes up. We can easily determine what fraction the arc is by finding what fraction its arc measurement is of the total circle (360).

Theorem 7-14 Arc Length

The length of an arc of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the circumference of the circle: length of $\overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360} \bullet 2\pi r$.

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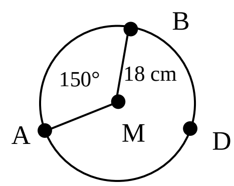
Congruent arcs

To try to make the difference between arc measure and arc length a bit clearer, think about this. It is possible for two arcs to have the same measure but different lengths. For instance in two circles with different radii (6 *in* and 12 *in*) each can have arc of 60°, but the length of these two arcs will be different because the circumference of each circle is different.

<u>Congruent arcs</u> are arcs that have the same measure and length. Another way of saying this is they have the same measure and are in the same circle or congruent circles.

Arc length example

Find the length of $\overset{\frown}{ADB}$ in $\odot M$ in terms of π.



$$m\overset{\frown}{ADB} = 360 - 150 = 210$$

$$len\,\overset{\frown}{ADB} = \frac{m\overset{\frown}{ADB}}{360} 2\pi r = \frac{210}{360} 2\pi 18 = \frac{7}{12} 36\pi = 7 \bullet 3\pi = 21\pi cm$$

Homework Assignment

p. 389 #1-39 odd, 42-47, 49-53, 55-59, 61-66